BFV data for EH gravity

Michele Schiavina, University of Pavia

UNIVERSITÀ DI PAVIA

Einstein–Hilbert gravity

BFV data

For $\phi \in C^\infty(\Sigma)$ and $X \in \mathfrak{X}(\Sigma)$, we have the constraint functions

$$\mathbb{H}_n(\phi) := \int\limits_\Sigma \left\{ \frac{1}{\mathrm{vol}_h} \left(\mathrm{Tr}_h[\Pi^2] - \frac{1}{d-1}[\mathrm{Tr}_h \Pi]^2 \right) + \mathrm{vol}_h R(h) \right\} \phi, \quad \mathbb{H}_\partial(X) := \int\limits_\Sigma \langle \Pi, \mathcal{L}_X h \rangle.$$

BFV fields: $\mathcal{F}^{(1)}_{EH} := T^*\left(\mathcal{R}(\Sigma) \times \mathfrak{X}[1](\Sigma) \times C^\infty[1](\Sigma)\right) \ni (h, \Pi, \xi^n, \xi^\partial, \chi_n, \chi_\partial)$,
with $\xi^n \in C^\infty[1](\Sigma), \xi^\partial \in \mathfrak{X}[1](\Sigma)$,
and $\chi_n \in C^\infty[-1](\Sigma) \otimes \mathrm{Dens}(\Sigma)$, $\chi_\partial \in \Omega^1[-1](\Sigma) \otimes \mathrm{Dens}(\Sigma)$.

$$\mathcal{S}^{(1)}_{EH} = \mathbb{H}_n(\xi^n) + \mathbb{H}_\partial(\xi^\partial) + \int\limits_\Sigma \chi_n \mathcal{L}_{\xi^\partial} \xi^n + \langle \chi_\partial, \xi^n \, \mathrm{grad}_h \xi^n \rangle + \frac{1}{2} \langle \chi_\partial, [\xi^\partial, \xi^\partial] \rangle.$$

BFV operator reads:

$$Q(\xi^n) = \mathcal{L}_{\xi^\partial} \xi^n, \qquad Q(\xi^\partial) = \xi^n \, \mathrm{grad}_h \xi^n + \frac{1}{2}[\xi^\partial, \xi^\partial]$$

$$Q(h) = \frac{\delta \mathbb{H}_n(\xi^n)}{\delta \Pi} - \mathcal{L}_{\xi^\partial} h, \qquad Q(\Pi) = -\frac{\delta \mathbb{H}_n(\xi^n)}{\delta h} - \mathcal{L}_{\xi^\partial} \Pi - (\chi_\partial \otimes_s d\xi^n)^{\sharp\sharp} \xi^n$$

$$Q(\chi_\partial) = \frac{\delta \mathbb{H}_\partial(\xi^\partial)}{\delta \xi^\partial} + \mathcal{L}_{\xi^\partial} \chi_\partial - \chi_n d\xi^n, \quad Q(\chi_n) = \frac{\delta \mathbb{H}_n(\xi^n)}{\delta \xi^n} + \mathcal{L}_{\xi^\partial} \chi_n - 2\mathcal{L}_{\chi^\sharp}(\xi^n \mathrm{vol}_h^{-\frac{1}{2}}) \mathrm{vol}_h^{\frac{1}{2}},$$

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Observations

The Q manifold $(\mathcal{F}^{(1)}_{EH}, Q^{(1)}_{EH})$ defines an L_∞-algebroid:

$$\mathcal{F}^{(1)}_{EH} \to \underbrace{T^*\mathcal{R}(\Sigma)}_{(h,\Pi)} \times \underbrace{\Omega^1[-1](\Sigma) \otimes \mathrm{Dens}(\Sigma)}_{\chi_\partial} \times \underbrace{C^\infty[-1](\Sigma) \otimes \mathrm{Dens}(\Sigma)}_{\chi_n}.$$

Resolution of the coisotropic locus of constraints
$C_{EH} = \{\mathbb{H}_n(\phi) = \mathbb{H}_\partial(X) = 0\} \subset T^*\mathcal{R}(\Sigma)$ [ADM, deWitt, ...] .
$Q^{(1)}_{EH}$ not "sum" of Lie algebroid differential & Koszul differential!
Due to multianchor term. $Q^{(1)}_{EH}$ defines ternary brackets on sections, which vanish only on *constant* sections but not on field-dependent gauge transformations.

Different from (most) gauge theories! Algebroid only on-shell.
Hamiltonian system with *on shell* symmetries. Momentum map?

L_∞ structure

Quadratic dependency in the fibres due to the term $(\chi_\partial \otimes_s d\xi^n)^{\sharp\sharp}\xi^n$,
Multi-anchor map, which vanishes **constant sections** except:

$$\rho^{(2)}((f_1, X_1), (f_2, X_2))(\Pi) = \chi_\partial^\sharp \otimes_s (f_1 \mathrm{grad}_h f_2 - f_2 \mathrm{grad}_h f_1), \tag{1a}$$

Multianchors require higher brackets. However, no cubic terms in the ξ variables in Q_{BFV}^{EH}. $\Rightarrow$ no higher brackets on **constant sections**.
From the defining Leibniz rule for three-brackets in L_∞ algebroids, denoting constant sections by s_i, we get that

$$[s_1, s_2, fs_3]_{(3)} = \rho^{(2)}(s_1, s_2)(f)s_3 + f[s_1, s_2, s_3] = \rho^{(2)}(s_1, s_2)(f)s_3.$$

Three-brackets vanish only on constant sections.
Jacobi identity on shell!
Akin to action algebroid for an abelian Lie algebra $\mathfrak{g}$. Here, the bracket of constant sections is zero. On the other hand, unless the action is trivial, the bracket is not identically zero on nonconstant sections.